CSE311 Microwave Engineering

LEC (09)
Smith Chart – Part I

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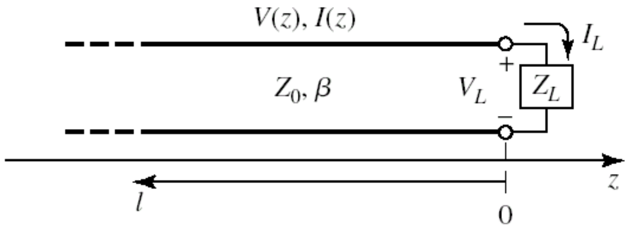
3.9 Smith Chart Analysis

3.9.1 Smith Chart Description

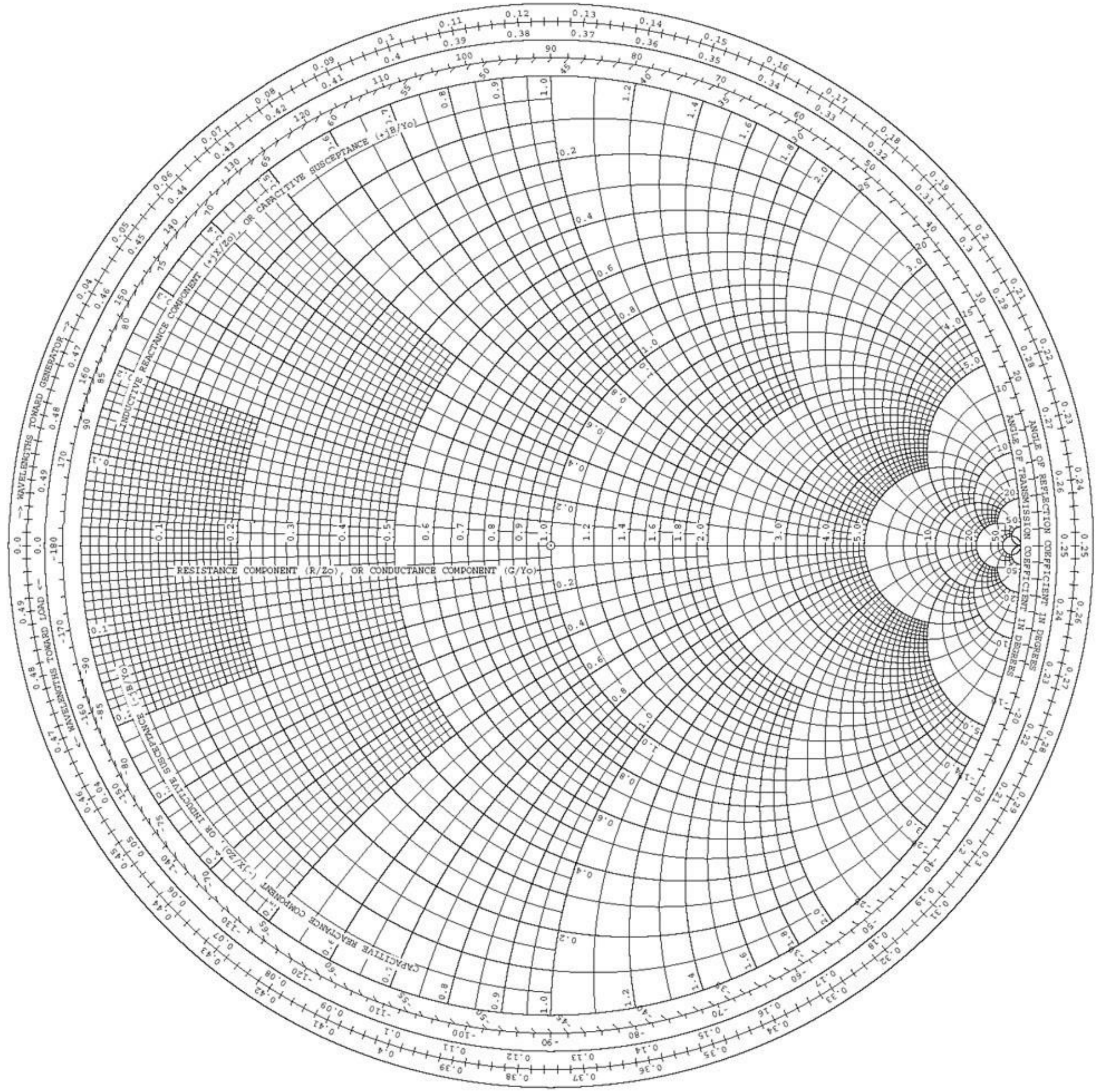
- The Smith chart shown in Fig. 3. 15 is a graphical aid that is very useful when solving transmission line problems.
- It was developed in 1939 by P. Smith at the Bell Telephone Lab.
- The Smith chart is more than a graphical just a graphical technique. Besides being an integral part of much recent Computer Aided Design (CAD) software and test equipment for microwave design, The Smith chart provides an extremely useful way of visualizing transmission line phenomenon.
- The microwave engineer can develop intuitions about transmission line and impedance matching problems.
- The basic relationship upon which the chart is constructed is:

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} \qquad (3.67)$$



- All values of resistance and reactance on the Smith chart are normalized, meaning that the load impedance to be plotted is divided by the characteristic impedance of the line ($z_L = Z_L / Z_o$). Let us identify the normalized impedance as:

$$z_L = \frac{Z_L}{Z_o} = \frac{R_L + jX_L}{Z_o} = r_L + jx_L \qquad (3.68)$$

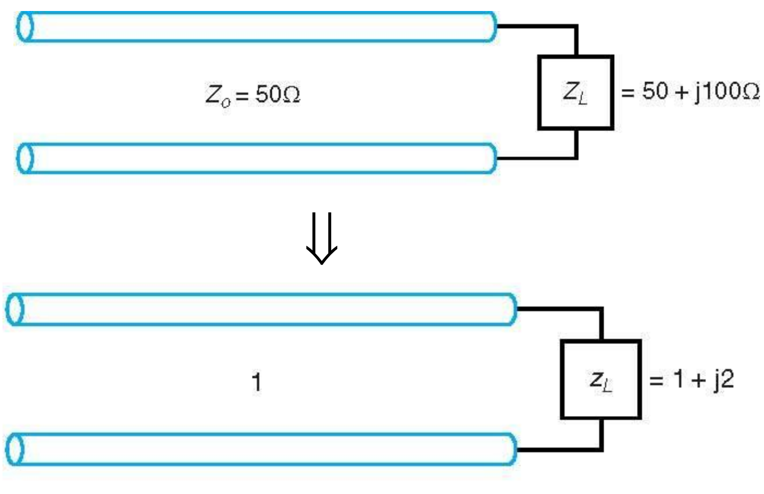


RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—

3.9 Smith Chart Analysis (Continued)

3.9.1 Smith Chart Description

- The two families of circles both appear on the Smith chart, as in Fig. 3.18. The resistance and reactance circles are orthogonal.
- For example, if a Z_o = 50 Ω transmission line is terminated in a load: Z_L = 50 + j100 Ω as shown. To locate this point on Smith Chart, normalize the load impedance, $z_L = Z_L/Z_o$ gives z_L = 1 + j2 Ω.
- The normalized load impedance is located at the intersection of the r =1 circle and the x =+2 circle.



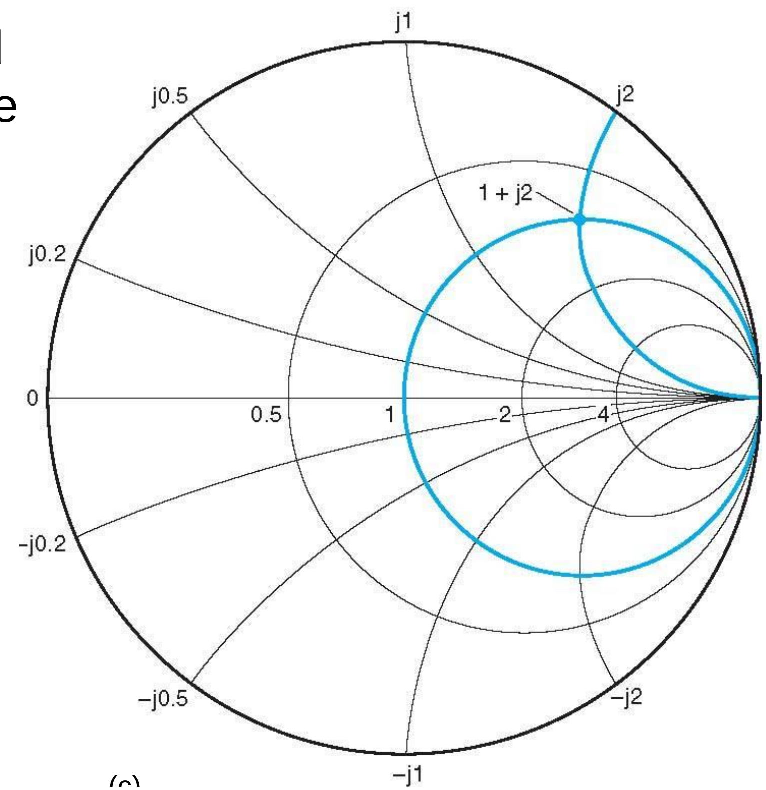


Fig. 3.18The Smith chart contains the constant-r circles and constant-x circles.

3.9 Smith Chart Analysis (Continued)

3.9.1 Smith Chart Description

Notes:

1. The Smith chart Fig. 3.15 has two circular scales on its outer edge. One is calibrated in fractional wavelength; the other is in degrees.
2. For convenience two scales are given. One showing an increase in distance for clockwise movement (Wavelength Toward Generator “WTG”) and the an increase for counterclockwise travel (Wavelength Toward Load “WTL).
3. The wavelength scale shows that a revolution on the chart is equivalent to a half wavelength (0.5 λ), and the movement clockwise or counterclockwise represents values of ℓ / λ.
4. The zero point of the WTG scale is rather arbitrary located to the left.
5. The degree scale shows that a complete revolution of the chart the reflection coefficient Γ goes through a complete cycle of 180^o positive and 180^o negative.
6. These scales are important and used to determine the impedance at various points along a line after the impedance is determined for any one specific point.
7. The radially scaled parameters are found on scales located directly below the circular part of the Smith chart. These include Voltage Standing Wave Ratio (VSWR), voltage reflection coefficient ($|\Gamma|$), power reflection coefficient ($|\Gamma|^2$), transmission coefficient (T), reflection loss, return loss (RL), conversion from coefficient ratios into dB, and the like. Note the most frequently used parameters are found to the left of center of the scales.

3.9 Smith Chart Analysis (Continued)

3.9.2 Smith Chart Application

The Smith chart is used to solve many of the transmission line problems which are illustrated by the following examples. These problems include:

1. Finding Voltage Standing Wave Ratio (VSWR) for a given Z_L
2. Finding the reflection coefficient (Γ) for a given Z_L.
3. Finding transmission coefficients (T) for a given Z_L.
4. Finding the admittance value (Y_L) for a given Z_L.
5. Finding the input impedance Z_{in} for a different loads (short circuit, open circuit, or any real or complex load Z_L).
6. Matching line terminations to the line using a $\lambda/4$ transformer.
7. Matching line terminations to the line using a single stub tuner.

For $Z_0 = 50\ \Omega$, so:

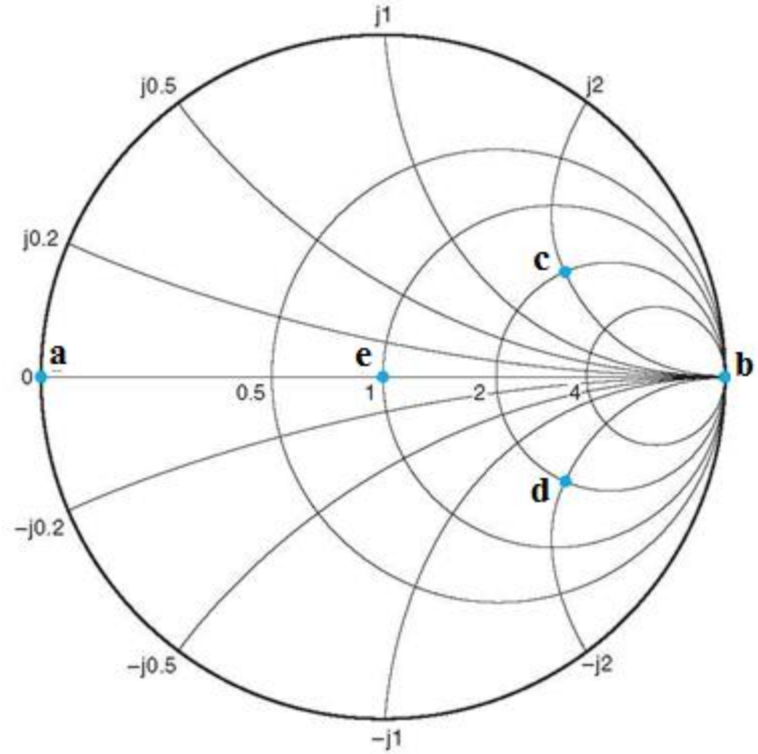
a → $Z_L = 0$ (short circuit)

b → $Z_L = \infty$ (open circuit)

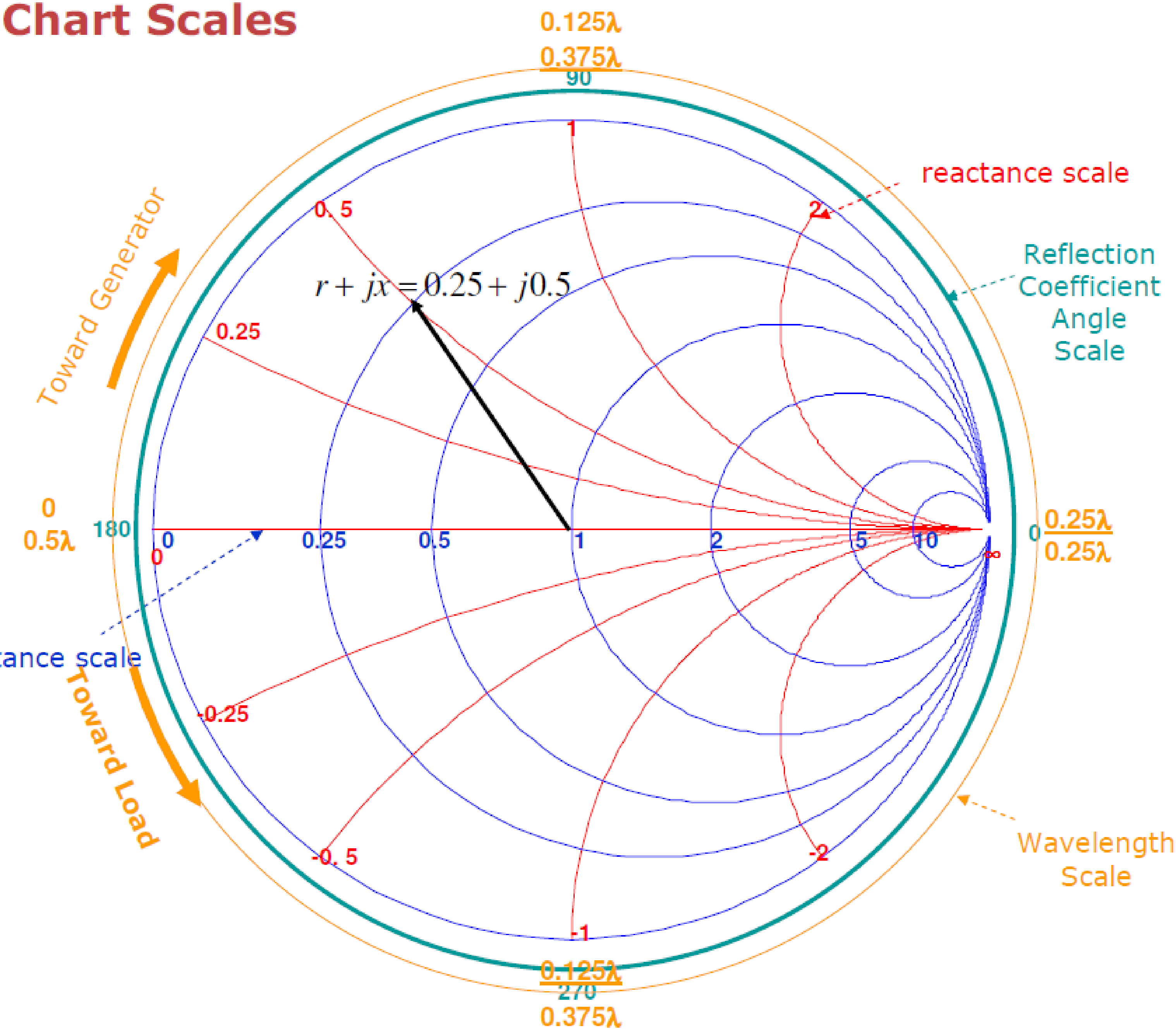
c → $Z_L = 100 + j100\ \Omega$

d → $Z_L = 100 - j100\ \Omega$

e → $Z_L = 50\ \Omega$



Smith Chart Scales



Example .1 (Calculating Reflection Coefficient)

If Z_L=100-j50 Ω, & Z_o=50 Ω

– Find Γ_L?

Ans:.

1st Normalize the Value of Z_L

$$z_L = (100 - j50)/50$$

$$z_L = 2 - j1$$

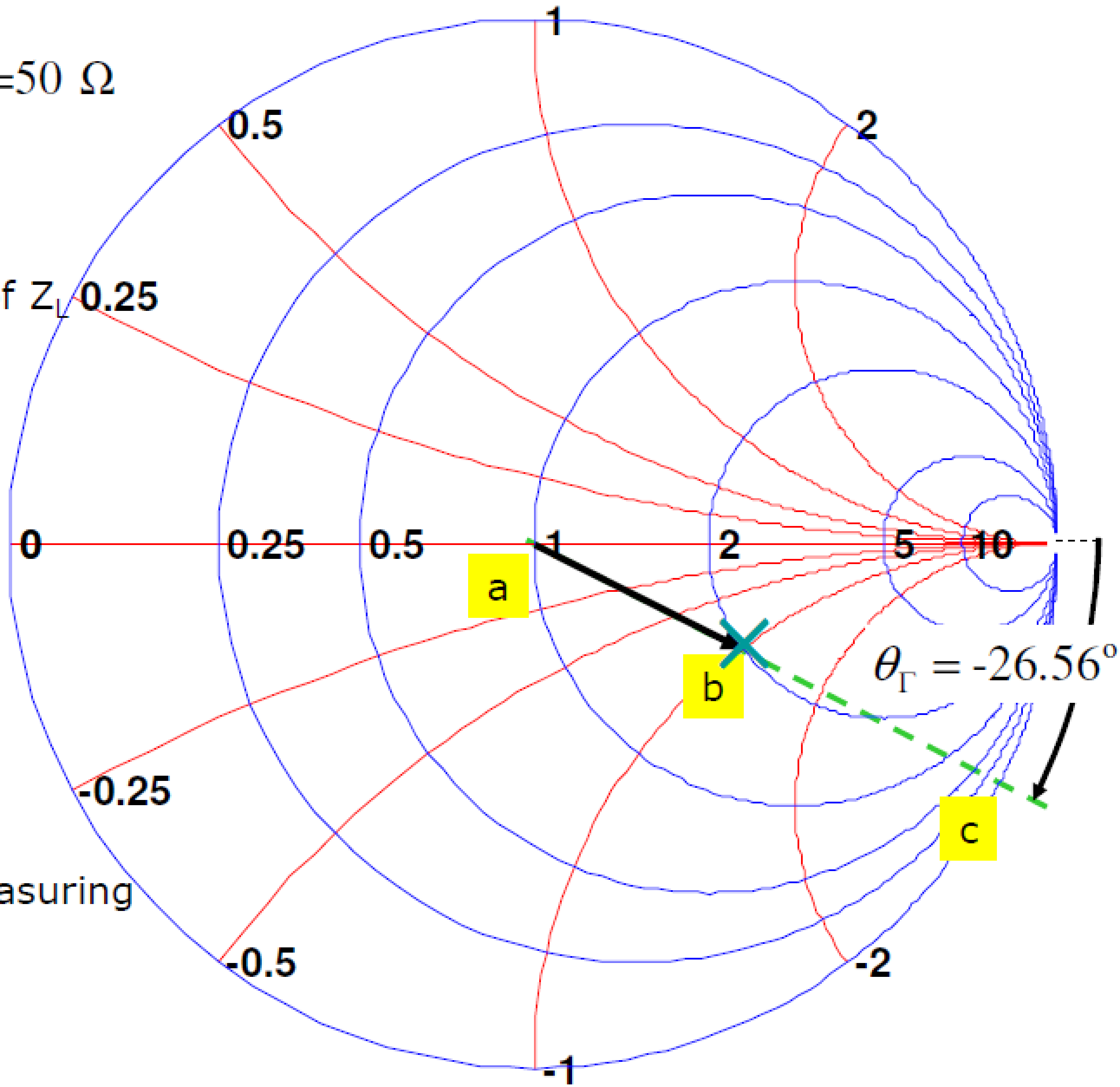
2nd Locate Z_L on SC

3rd Draw a line from the center of SC passing by Z_L until the edge of SC

4th Use the angle scale to determine the value of θ_L.

5th Calculate $|\Gamma_L|$, by measuring

$$|\Gamma_L| = \frac{|\vec{r}_{ab}|(cm)}{|\vec{r}_{ac}|(cm)} = 0.447$$



3.9 Smith Chart Analysis (Continued)

3.9.2 Smith Chart Application

(1) VSWR Determination

Example 3.11

Determine the Voltage Standing Wave Ratio (VSWR) of a transmission line that result when its load impedance Z_L = 50 + j 50 Ω and its characteristic impedance of the Z_o = 50 Ω.

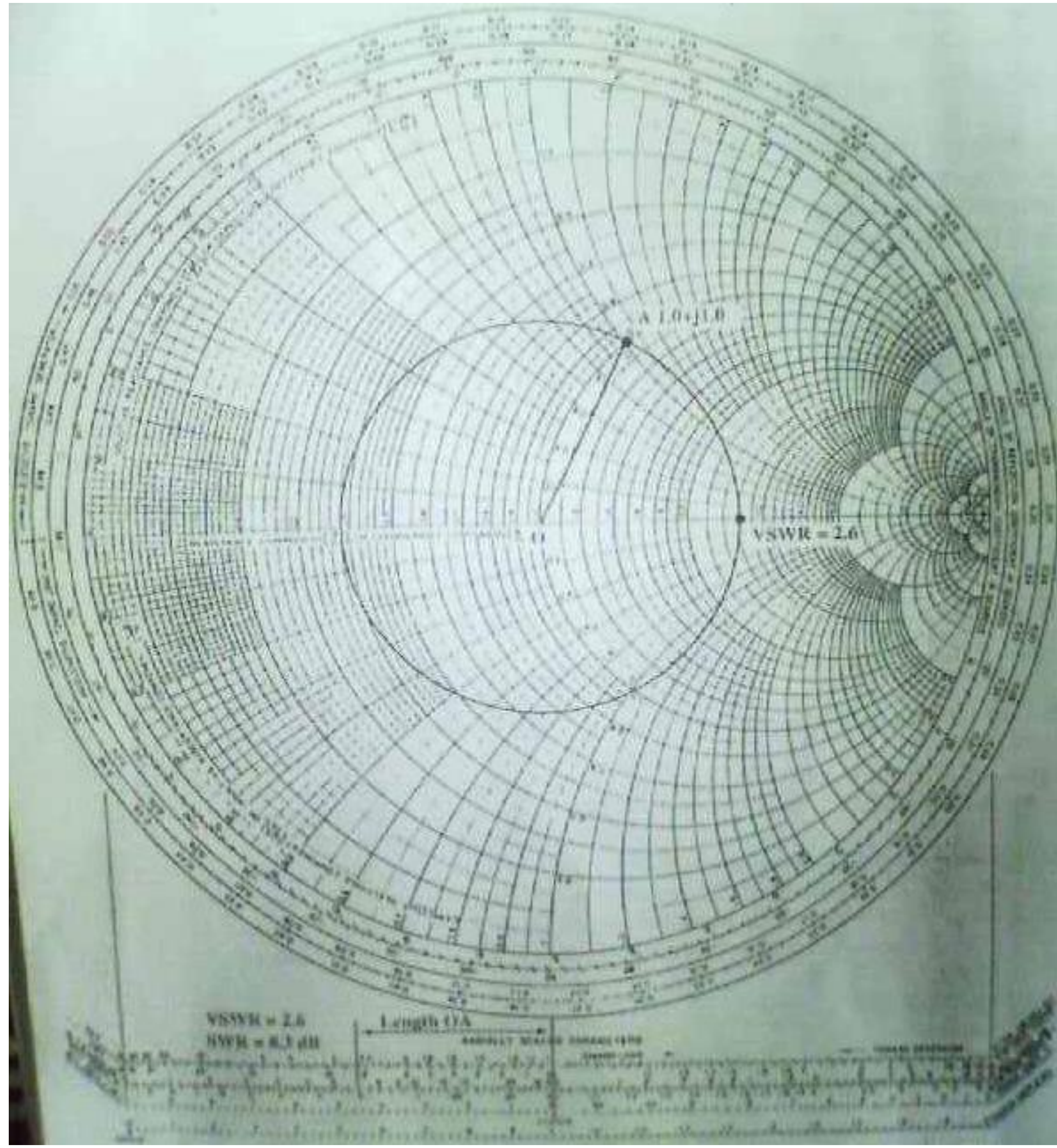
Solution

1. Normalize Z_L (divide Z_L by Z_o) then $z_L = Z_L / Z_o$ = (50 + j 50)/ 50 = 1 + j1.
2. Plot z_L on the Smith chart (the point of intersection of the circle r = 1 and the arc marked x =1). This the point A in Fig. 3. 19.
3. With a compass, draw a circle centered at 1 + j0 having a radius equal to the distance from the center to the point A. This circle is called the constant VSWR circle. All impedances on this circle produce the same VSWR.
4. Read the value of the VSWR where the circle cross the diameter line to the right of the center (VSWR = 2.6). An alternate method is to set compass for the distance of the radius OA, then transfer this distance to the radially scaled parameter directly below the chart. Place one end of the compass on the center point of the SWR scale and the other end of the compass to the left on the SWR scale. The numerals above the SWR line indicate the VSWR, while the numerals below the line indicate the SWR in dB. Read the VSWR directly off the scale (VSWR = 2.6 or SWR_{dB} = 8.3 dB) as shown in Fig. 3. 19.

3.9 Smith Chart Analysis (Continued)

3.9.2 Smith Chart Application; (1) VSWR Determination; *Example 3.11 Solution*

Fig. 3.19 Determination of VSWR using the Smith chart



3.9 Smith Chart Analysis (Continued)

3.9.2 Smith Chart Application

(2) Reflection Coefficient (Γ) and Transmission Coefficient (T) Determination

Example 3.12

Determine the reflection coefficient (Γ) of a transmission line that result when its load impedance Z_L = 100 - j 200 Ω and its characteristic impedance of the Z_o=100 Ω.

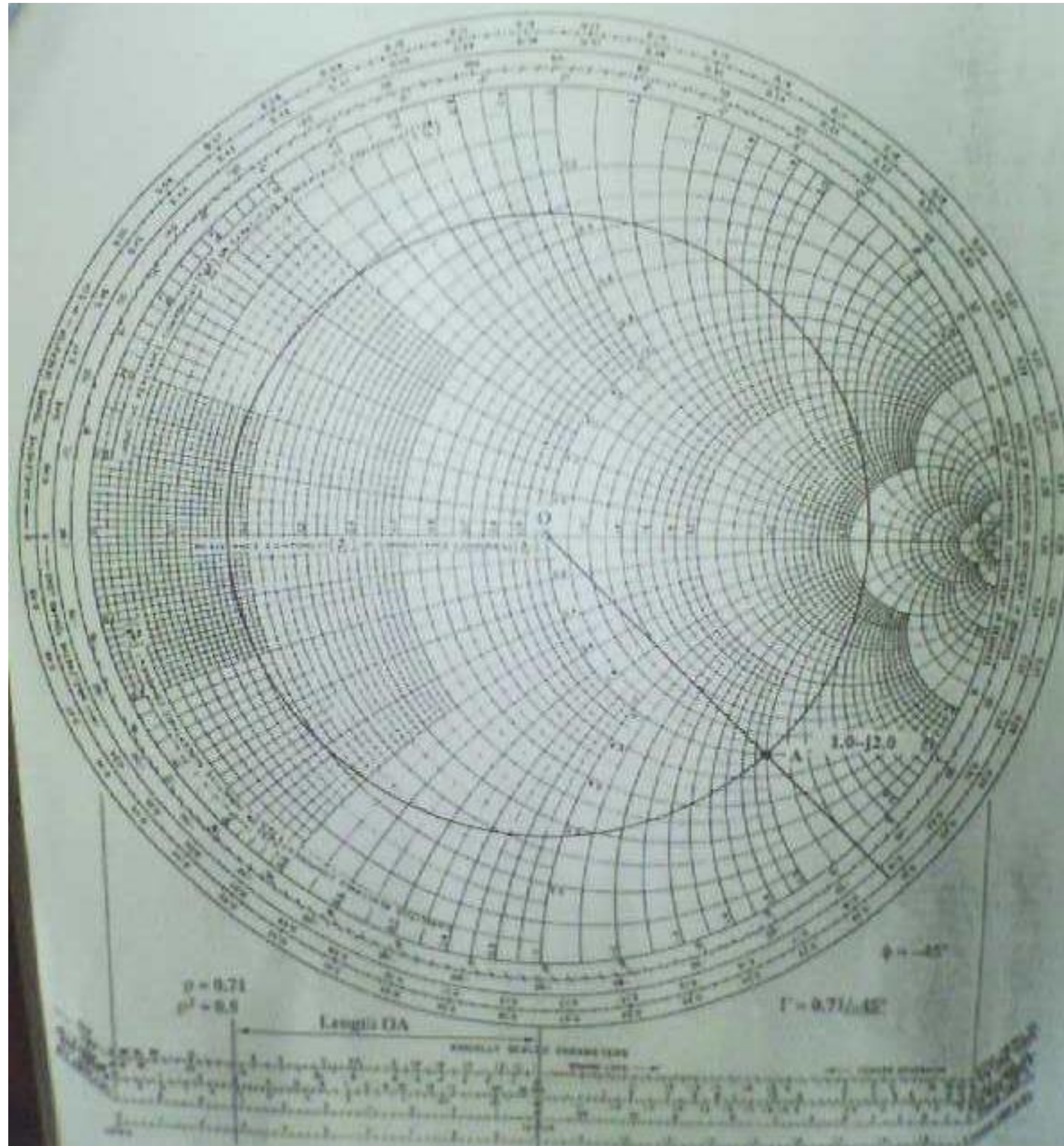
Solution

1. Normalize Z_L (divide Z_L by Z_o) then $z_L = Z_L / Z_o$ = (100 - j 200)/ 100 = 1 - j2 .
2. Plot z_L on the Smith chart (the point of intersection of the circle r = 1 and the arc marked x = -2). This the point A in Fig. 3. 20. Draw VSWR for this load.
3. Set compass for the distance of the radius OA, then transfer this distance to the Voltage Reflection Coefficient scale, on radially scaled parameters (This is labeled Refl. Coeff. E or I). Place one end of the compass on the center point of the and the other end of the compass to the left on the Reflection Coefficient scale. Read off the magnitude of Γ ($|\Gamma|$ = 0.71 and the power Reflection Coefficient $|\Gamma|^2 \approx 0.5$) as shown in Fig. 3.20.
4. To get the phase angle of Γ, draw a line that starts from the center of the VSWR circle (1 + j0) and pass through point A extending outward to the angle of Reflection coefficient in Degrees scale at the edge of the chart. Read the phase angle of reflection coefficient where this line cross the angle value (φ= - 45^o).
5. Repeat the same steps for T, we get |T| = 0.71 and φ = -22.5^o.

3.9 Smith Chart Analysis (Continued)

3.9.2 Smith Chart Application; (2) Γ and T Determination; *Example 3.12 Solution*

Fig. 3.20 Determination of voltage reflection and transmission coefficients using the Smith chart.



3.9 Smith Chart Analysis (Continued)

3.9.2 Smith Chart Application

(3) Admittance (Y_L) Determination

Example 3.13

Determine the admittance Y_L that result when the load impedance Z_L= 50 + j100 Ω and the transmission line characteristic impedance is Z_o = 50 Ω.

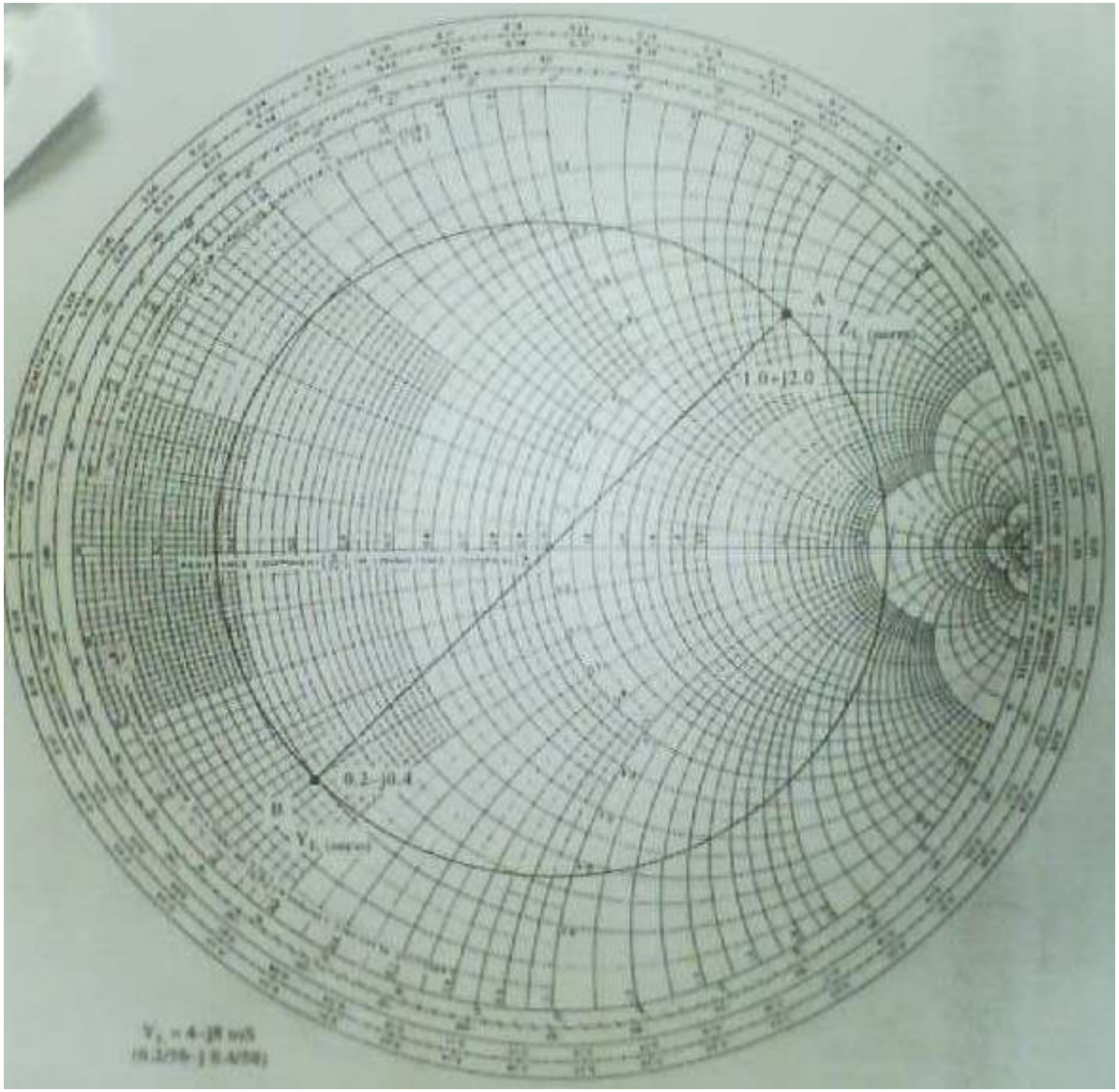
Solution

1. Normalize Z_L (divide Z_L by Z_o) then $z_L = Z_L / Z_o$ = (50 + j 100)/ 50 = 1 + j2 .
2. Plot z_L on the Smith chart (the point of intersection of the circle r = 1 and the arc marked x =2). This the point A in Fig. 3. 21.
3. Draw VSWR for this load.
4. Extend the line AO diametrically opposite to meet the VSWR at point B.
5. Read the normalized impedance at B which is the admittance at A Then y_L = 0.2 –j 0.4 and $Y_L = y_L \times Y_o$ = = $y_L \times Y_o$ (0.2 –j 0.4) x 0.02 = 4 – j 8 mS.
6. Point B is λ/4 away from point A as shown in Fig. 3. 21.

3.9 Smith Chart Analysis (Continued)

3.9.2 Smith Chart Application; (3) Y_L Determination; *Example 3.13 Solution*

Fig. 3.21 Admittance determination using the Smith chart



3.9 Smith Chart Analysis (Continued)

3.9.2 Smith Chart Application

(4) Input Impedance (Z_{in}) Determination

Example 3.14

Determine the input impedance Z_{in} of a transmission line at 0.2 λ from the load when the load impedance Z_L= 75 +j75 Ω and the transmission line characteristic impedance is Z_o = 50 Ω.

Solution

1. Normalize Z_L (divide Z_L by Z_o) then $z_L = Z_L / Z_o$ = (75 + j 75)/ 50 = 1.5 + j1.5 .
2. Plot z_L on the Smith chart (the point of intersection of the circle r = 1.5 and the arc marked x =1.5). This the point A in Fig. 3. 22.
3. Draw VSWR for this load.
4. Extend the line OA outward the outer scale (WTG) and then read the relative λ value off this scale (relative λ = 0.194 λ toward generator).
5. From the relative position of the load rotate clockwise (toward generator) 0.2 λ.
6. Draw a line from the center of the chart through this point (Point B) crossing the VSWR circle.
7. Read the normalized impedance at point B as shown in Fig. 3.22.
   the normalized z_{in} = 0.46 – j 0.68 (0.2 λ from the load) and
   $Z_{in} = z_{in} \times Z_o$ = 23 - j 34 Ω.

3.9 Smith Chart Analysis (Continued)

3.9.2 Smith Chart Application; (4) Z_{in} Determination; *Example 3.14 Solution*

Fig. 3.22 Input impedance (Z_{in}) determination using the Smith chart